Rigorous Quasi-TEM Analysis of Multiconductor Transmission Lines in Bi-Isotropic Media— Part I: Theoretical Analysis for General Inhomogeneous Media and Generalization to Bianisotropic Media

Frank Olyslager, *Member, IEEE*, Eric Laermans, and Daniël De Zutter, *Member, IEEE*

***Abstract*— The quasi-TEM approximation for multiconductor lines embedded in inhomogeneous bi-isotropic media is developed. It is shown that in the quasi-TEM limit a multiconductor line in bi-isotropic media can be represented by a coupled set of nonreciprocal and nonsymmetric circuit transmission lines. The effect on the circuit parameters of absence of losses, reciprocity and symmetry properties of the geometry and of the equations is investigated. Finally, the generalization to full bianisotropic materials is studied.**

I. Introduction

THE last few years the interest in chiral materials and more general in bianisotropic media has grown and is still growing rapidly. There have been made considerable advances not only in setting up the theoretical framework but also in the manufacturing of these materials and in their application in new devices [1]. Interesting properties of these materials for use in for example waveguiding structures [2], microstrip antenna arrays [3], and absorbing and nonreflecting coatings [4] and [5] have been studied.

Multiconductor lines in isotropic media have been studied rigorously in the past in the full-wave and the quasi-TEM regime. For an overview we refer to [6]. In [7] and [8] the quasi-TEM approximation of multiconductor lines in isotropic media was studied from a theoretical point of view. It was shown that in the quasi-TEM limit such a multiconductor line can be represented in circuit terms by a set of coupled transmission lines described by the classical telegraphers equations. Numerical analyses, mostly with integral equations and for layered media, can be found for example in [9], [10] and for the anisotropic case in [11].

In the present contribution we will first concentrate on the analysis of multiconductor lines embedded in bi-isotropic media. First, we will generalize the results of [7] from isotropic to inhomogeneous bi-isotropic media and later to bianisotropic media. We will show that in the quasi-TEM limit a multiconductor line embedded in general bi-isotropic and bianisotropic media can be represented by a nonreciprocal and nonsymmetric uniform set of coupled circuit transmission lines. In the sequel we will call these lines bitransmission lines. To our knowledge the first paper to deal with coupled bitransmission lines to represent the propagation in anisotropic chiral media was [12]. A more advanced analysis of a single nonreciprocal transmission line to represent the propagation in a nonreciprocal waveguide was discussed in [13]. Both [12] and [13] deal with the fullwave regime where the transmission line model is only an approximation, which does not follow from the Maxwell equations, for the true propagation in the waveguide. An elementary quasi-TEM approximation of a single microstrip line on a nonreciprocal bi-isotropic substrate was presented in [14]. In [15] a detailed study on circuit level was made of a single bitransmission line and the equivalence between the bitransmission line and the propagation of circulary polarized plane waves in layered bi-isotropic media was pointed out.

Secondly, we will also investigate a number of properties of the circuit parameter matrices characterising these bitransmission lines. Especially we will investigate the effects of the absence of losses, of reciprocity, and of symmetry properties of the equations and geometry. Concerning the reciprocity we will point that the nomenclature in [15] is somewhat unfortunate.

A quasi-TEM analysis is often regarded as an approximation for *low frequencies* and it is known that at *low frequencies* the chirality becomes proportional to frequency [16]. However one should keep in mind that the term *low frequency* was used in two different meanings in the previous sentence. Low frequency for the quasi-TEM limit means that the dimensions of the cross section of the structure are small compared to the wavelength [8], [17]. In fact the quasi-TEM analysis neglects wave propagation effects in the cross section of the structure and only takes propagation in the propagation direction into account. This means that low frequency for the quasi-TEM limit does not necessary mean low frequency for the materials. One can still work at frequencies for the material parameters where chirality is not necessary proportional to frequency

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The authors are with the Electromagnetics Group, Department of Information Technology, University of Ghent, 9000 Ghent, Belgium.
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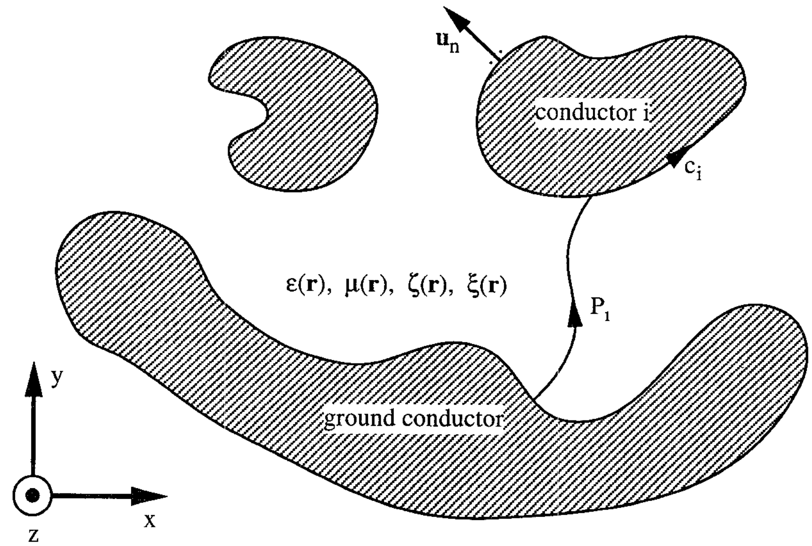


Fig. 1. Geometry of a multiconductor line in an inhomogeneous bi-isotropic background medium.

and where there can exist nonreciprocity. Further it has been shown in the isotropic case [6] that the quasi-TEM analysis remains useful, at least qualitatively, to high frequencies for the fundamental modes. In fact one can just regard the quasi-TEM analysis as an approximation for the fullwave analysis where the longitudinal field components have been neglected.

A restriction of a quasi-TEM analysis is that it assumes that the materials are lossless. The composite bi-isotropic materials at microwave frequencies presently constructed still contain dielectric and chiral losses [1]. With perturbation techniques, as is applied for the isotropic case in [18], it is easy to include dielectric, magnetic, chiral and conductor losses in the analysis.

In a last section we will generalise the analysis to full bianisotropic materials. In [19] a quasi-TEM analysis for anisotropic media was presented. We will also show for which bianisotropic media the analysis reduces to the bi-isotropic case. This is important because recently [20] it has been argued that nonreciprocal materials are inherently anisotropic or bianisotropic which would make a discussion of nonreciprocal bi-isotropic materials obsolete. However, as we will show, the analysis for some nonreciprocal bianisotropic media reduces to the analysis of nonreciprocal bi-isotropic materials in the quasi-TEM limit. This fact together with the fact that the full bi-isotropic case is not more complicated than the reciprocal bi-isotropic (i.e. chiral) case motivates the use of general bi-isotropic materials in the sequel. It will also be shown that the reciprocal bi-isotropic problem, which is a complex problem, can be reduced to the solution of a nonreciprocal bi-isotropic problem, which is real.

The theory and results of the present paper and an accompanying paper [21], dealing with the numerical solution, have been announced in [22]. In [21] the potential problem constructed in this paper is solved numerically with the method of moments and the pointmatching technique for conductors with general cross section embedded in layered bi-isotropic media. [21] also contains a number of examples illustrating the properties of the circuit parameters derived in this paper.

II. THE POTENTIAL PROBLEM

Consider the waveguiding structure of Fig. 1 which is invariant in the longitudinal z-direction and inhomogeneous in the transversal cross section. The waveguide consist of a number of PEC conductors with arbitrary cross section embedded in an inhomogeneous bi-isotropic background characterized by the following constitutive relations

$$\begin{aligned} \boldsymbol{D} &= \varepsilon(\boldsymbol{r})\boldsymbol{E} + \xi(\boldsymbol{r})\boldsymbol{H} \\ \boldsymbol{B} &= \zeta(\boldsymbol{r})\boldsymbol{E} + \mu(\boldsymbol{r})\boldsymbol{H} \end{aligned} \tag{1}$$

with $\boldsymbol{r} = x\boldsymbol{u}_x + y\boldsymbol{u}_y$. For our purposes we write (1) in a more suitable form which corresponds to the form used in [23]

$$\begin{aligned} \boldsymbol{D} &= \frac{n^2(\boldsymbol{r})}{\mu(\boldsymbol{r})}\boldsymbol{E} + \frac{\xi(\boldsymbol{r})}{\mu(\boldsymbol{r})}\boldsymbol{B} \\ \boldsymbol{H} &= -\frac{\zeta(\boldsymbol{r})}{\mu(\boldsymbol{r})}\boldsymbol{E} + \frac{1}{\mu(\boldsymbol{r})}\boldsymbol{B} \end{aligned} \tag{2}$$

where $n^2(\boldsymbol{r}) = \varepsilon(\boldsymbol{r})\mu(\boldsymbol{r}) - \zeta(\boldsymbol{r})\xi(\boldsymbol{r})$. It is assumed that the bi-isotropic medium is lossless, i.e. that $\varepsilon(\boldsymbol{r}) = \varepsilon_r(\boldsymbol{r})\varepsilon_0$ and $\mu(\boldsymbol{r}) = \mu_r(\boldsymbol{r})\mu_0$ are real and that $\zeta(\boldsymbol{r}) = \xi(\boldsymbol{r})^*$ (see [16]). If we write ζ as $(\chi + j\kappa)\sqrt{\varepsilon_0\mu_0}$ and ξ as $(\chi - j\kappa)\sqrt{\varepsilon_0\mu_0}$, with $\chi(\boldsymbol{r})$ the Tellegen parameter or nonreciprocity parameter and $\kappa(\boldsymbol{r})$ the chirality parameter, then absence of losses means that also $\kappa(\boldsymbol{r})$ and $\chi(\boldsymbol{r})$ are real. One of the conductors is chosen as reference or ground conductor. It is allowed that parts or the whole of this ground conductor are located at infinity. The other conductors have finite dimensions and are numbered from 1 to N. An arbitrary line which connects conductor i with the ground conductor is denoted P_i and c_i denotes the boundary curve of this conductor. Finally $\boldsymbol{u}_n$ denotes a unit vector in the cross section perpendicular to the conductors.

We are looking for solutions of the Maxwell equations with $\exp(j\omega t)$ $\exp(-j\beta z)$ dependence. The fields, sources and propagation coefficient β are expanded in a Taylor series of the pulsation ω

$$\begin{aligned} \boldsymbol{E}_{\mathrm{tr}} &= \boldsymbol{E}_{\mathrm{tr},0} + \omega \boldsymbol{E}_{\mathrm{tr},1} + \cdots & E_z &= E_{z,0} + \omega E_{z,1} + \cdots \\ \boldsymbol{H}_{\mathrm{tr}} &= \boldsymbol{H}_{\mathrm{tr},0} + \omega \boldsymbol{H}_{\mathrm{tr},1} + \cdots & H_z &= H_{z,0} + \omega H_{z,1} + \cdots \\ \boldsymbol{D}_{\mathrm{tr}} &= \boldsymbol{D}_{\mathrm{tr},0} + \omega \boldsymbol{D}_{\mathrm{tr},1} + \cdots & D_z &= D_{z,0} + \omega D_{z,1} + \cdots \\ \boldsymbol{B}_{\mathrm{tr}} &= \boldsymbol{B}_{\mathrm{tr},0} + \omega \boldsymbol{B}_{\mathrm{tr},1} + \cdots & B_z &= B_{z,0} + \omega B_{z,1} + \cdots \\ \boldsymbol{J}_{\mathrm{tr}} &= \boldsymbol{J}_{\mathrm{tr},0} + \omega \boldsymbol{J}_{\mathrm{tr},1} + \cdots & J_z &= J_{z,0} + \omega J_{z,1} + \cdots \\ \rho &= \rho_0 + \omega \rho_1 + \cdots & \beta &= \beta_0 + \omega \beta_1 + \cdots \end{aligned} \quad (3)$$

The subscript 'tr' denotes transversal components. $\boldsymbol{J}$ and ρ are respectively the surface current densities and surface charge densities on the conductor surfaces. Since the structure is lossless and behaves fully TEM when $\omega \to 0$ we have that $\beta_0 = E_{z,0} = H_{z,0} = D_{z,0} = B_{z,0} = 0$ and $\boldsymbol{J}_{\mathrm{tr},0} = \boldsymbol{0}$ (see also [7]). If these expansions are inserted in the Maxwell equations and if terms of the same order in ω are identified then one obtains, amongst others, the following equations

$$\begin{aligned} \nabla_{\mathrm{tr}} \times \boldsymbol{E}_{\mathrm{tr},0} &= \boldsymbol{0} \\ \nabla_{\mathrm{tr}} \cdot \boldsymbol{D}_{\mathrm{tr},0} &= 0 \\ \nabla_{\mathrm{tr}} \times \boldsymbol{H}_{\mathrm{tr},0} &= \boldsymbol{0} \\ \nabla_{\mathrm{tr}} \cdot \boldsymbol{B}_{\mathrm{tr},0} &= 0 \\ \nabla_{\mathrm{tr}} \times E_{z,1}\boldsymbol{u}_z - j\beta_1 \boldsymbol{u}_z \times \boldsymbol{E}_{\mathrm{tr},0} &= -j\boldsymbol{B}_{\mathrm{tr},0} \\ \nabla_{\mathrm{tr}} \times H_{z,1}\boldsymbol{u}_z - j\beta_1 \boldsymbol{u}_z \times \boldsymbol{H}_{\mathrm{tr},0} &= j\boldsymbol{D}_{\mathrm{tr},0}. \end{aligned} \quad (4)$$

If on the other hand the expansions are inserted in the boundary conditions at the conductors one obtains

$$\begin{aligned} \boldsymbol{u}_n \times \boldsymbol{E}_{\mathrm{tr},0} &= \boldsymbol{0} & \boldsymbol{u}_n \cdot \boldsymbol{D}_{\mathrm{tr},0} &= \rho_0 \\ \boldsymbol{u}_n \times \boldsymbol{H}_{\mathrm{tr},0} &= J_{z,0}\boldsymbol{u}_z & \boldsymbol{u}_n \cdot \boldsymbol{B}_{\mathrm{tr},0} &= 0 \\ \boldsymbol{u}_n \times \boldsymbol{B}_{\mathrm{tr},0} &= \mu J_{z,0}\boldsymbol{u}_z & \boldsymbol{u}_n \cdot \boldsymbol{E}_{\mathrm{tr},0} &= \frac{\mu\rho_0}{n^2} \end{aligned} \quad (5)$$

where the last two conditions follow from the other ones by using the constitutive relations (2).

The first respectively fourth equation of (4) and the first respectively fourth equation of (5) allows us to derive $\boldsymbol{E}_{\mathrm{tr},0}$ respectively $\boldsymbol{B}_{\mathrm{tr},0}$ from a scalar potential ϕ respectively ψ as follows:

$$\boldsymbol{E}_{\mathrm{tr},0}(\boldsymbol{r}) = -\nabla_{\mathrm{tr}}\phi(\boldsymbol{r}) \quad \boldsymbol{B}_{\mathrm{tr},0}(\boldsymbol{r}) = \nabla_{\mathrm{tr}} \times \psi(\boldsymbol{r})\boldsymbol{u}_z \quad (6)$$

ψ is also called the flux function and corresponds to the z-component $A_{z,0}$ of the vector potential. If these expressions, together with the constitutive relations (2), are combined with the second and third equations of (4) one obtains

$$\begin{aligned} -\nabla_{\mathrm{tr}} \cdot \frac{n^2}{\mu}\nabla_{\mathrm{tr}}\phi + \nabla_{\mathrm{tr}} \cdot \left(\frac{\xi}{\mu}\nabla_{\mathrm{tr}} \times \psi\boldsymbol{u}_z\right) &= 0 \\ \boldsymbol{u}_z \cdot \left[\nabla_{\mathrm{tr}} \times \left(\frac{\zeta}{\mu}\nabla_{\mathrm{tr}}\phi + \frac{1}{\mu}\nabla_{\mathrm{tr}} \times \psi\boldsymbol{u}_z\right)\right] &= 0. \end{aligned} \quad (7)$$

The boundary conditions for the potentials at the conductors are obtained from (5) and (6)

$$\begin{aligned} \phi &= \text{constant} & \psi &= \text{constant} \\ \frac{\partial\phi}{\partial n} &= -\frac{\mu}{n^2}\rho_0 & \frac{\partial\psi}{\partial n} &= -\mu J_{z,0}. \end{aligned} \quad (8)$$

The set (7) and the conditions (8) define a coupled potential problem for the potentials ϕ and ψ.

III. The Bitransmission Lines

Since the problem (7), (8) is a linear differential problem there will be a linear relation between the constant potentials of the conductors and the total surface charge and surface current on the conductors. The potentials ϕ and ψ are chosen to be zero on the ground conductor and the potentials of conductor $k(k = 1, 2, \cdots, N)$ are denoted ϕ_k and ψ_k. If Q_j and I_j are respectively the charge and current on conductor j then we can write

$$\begin{aligned} Q_j &= \oint_{c_i} \rho_0 \, dc = \sum_{k=1}^{N} a_{jk}\phi_k + \sum_{k=1}^{N} b_{jk}\psi_k \\ I_j &= \oint_{c_i} J_{z,0} \, dc = \sum_{k=1}^{N} c_{jk}\phi_k + \sum_{k=1}^{N} d_{jk}\psi_k \\ j &= 1, 2, \cdots, N \end{aligned} \quad (9)$$

where a_{jk}, b_{jk}, c_{jk} and d_{jk} are coefficients which follow from the solution of the potential problem (7), (8). If this system of equations is solved for Q_j and I_j and if a matrix formalism is used one finds with self explaining notations that:

$$\begin{aligned} \overline{Q} &= \overline{\overline{C}}\overline{\phi} + \overline{\overline{X}}\overline{I} \\ \overline{\Psi} &= \overline{\overline{Z}}\overline{\phi} + \overline{\overline{L}}\overline{I} \end{aligned} \quad (10)$$

with

$$\begin{aligned} \overline{\overline{C}} &= \overline{\overline{a}} - \overline{\overline{b}}\,\overline{\overline{d}}^{-1}\overline{\overline{c}} \\ \overline{\overline{X}} &= \overline{\overline{b}}\,\overline{\overline{d}}^{-1} \\ \overline{\overline{Z}} &= -\overline{\overline{d}}^{-1}\overline{\overline{c}} \\ \overline{\overline{L}} &= \overline{\overline{d}}^{-1} \end{aligned} \quad (11)$$

where for example $\overline{\overline{a}}$ is the matrix of the a_{jk} coefficients. The matrices $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are respectively the capacitance and inductance matrix and $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ are two new matrices which describe the coupling between the electric and magnetic problem. The circuit matrices are determined by solving the coupled potential problem (7) $2N$ times where each time another ϕ_k or ψ_k $(k = 1, 2, \cdots, N)$ is taken different from zero.

To construct the transmission line equations we use the last two equations of (4). As will be demonstrated these equations show that for a modal solution the potentials ϕ_i and ψ_i of the conductors are not independent and that the charges Q_i and currents I_i on the conductors are also coupled. Let us start by integrating the fifth equation of (4) along the path P_i

$$\begin{aligned} -\int_{P_i} \nabla_{\mathrm{tr}}E_{z,1} \cdot \boldsymbol{dl} + j\beta_1 \int_{P_i} \boldsymbol{E}_{\mathrm{tr},0} \cdot \boldsymbol{dl} \\ = -j\int_{P_i} (\boldsymbol{u}_z \times \boldsymbol{B}_{\mathrm{tr},0}) \cdot \boldsymbol{dl} \qquad i = 1, 2, \cdots, N. \end{aligned} \quad (12)$$

By using (6) and by taking into account the fact that $E_{z,1}$ vanishes on the conductors one finds

$$j\beta_1\phi_i = j\psi_i \qquad i = 1, 2, \cdots, N. \quad (13)$$

Similarly by integrating the last equation of (4) along the contour c_i and by using the second and third boundary

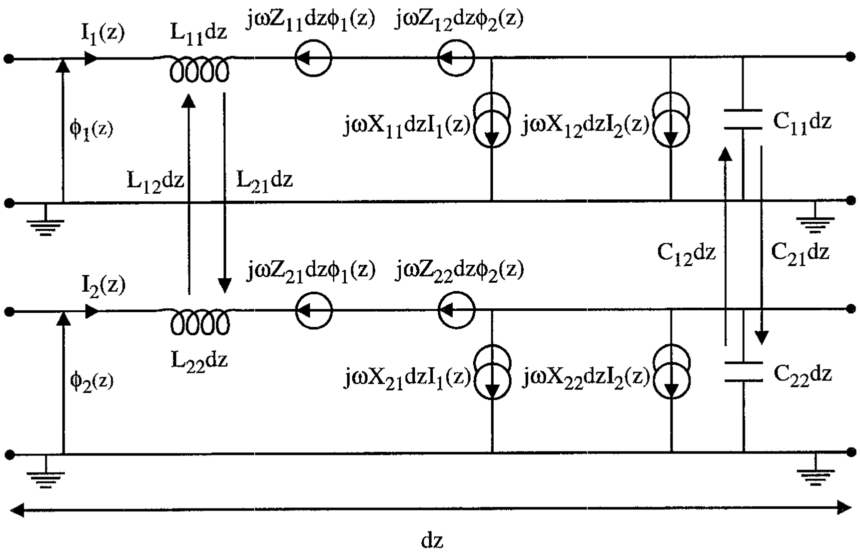


Fig. 2. Circuit representation of an infinitesimal section of two coupled bitransmission lines.

condition of (4) the following equation is obtained:

$$j\beta_1 I_i = jQ_i \qquad i = 1, 2, \cdots, N. \tag{14}$$

If (13) and (14) are multiplied by ω and combined with (10) and if $-j\omega\beta_1$ is replaced by d/dz—inverse spatial Fourier transform in the longitudinal direction—one finds the following coupled set of bitransmission line equations:

$$\begin{aligned}\frac{d\overline{\phi}(z)}{dz} &= -j\omega\overline{\overline{Z}}\overline{\phi}(z) - j\omega\overline{\overline{L}}\overline{I}(z)\\ \frac{d\overline{I}(z)}{dz} &= -j\omega\overline{\overline{C}}\overline{\phi}(z) - j\omega\overline{\overline{X}}\overline{I}(z).\end{aligned} \tag{15}$$

An infinitesimal segment of a classical set of transmission lines corresponds to a coupled LC circuit. For the bitransmission line of (15) a coupled voltage controlled voltage source and a coupled current controlled current source has to be added (see Fig. 2 for two-line case) as was shown for the single transmission line case in [12] and [15].

IV. Properties of $\overline{\overline{C}}, \overline{\overline{L}}, \overline{\overline{X}}$, and $\overline{\overline{Z}}$

In this section we will derive a number of properties of the $\overline{\overline{C}}, \overline{\overline{L}}, \overline{\overline{X}}$, and $\overline{\overline{Z}}$ matrices which are related to the material parameters and/or geometry of the structure of Fig. 1.

We used the quasi-TEM analysis to construct (15) which results into some special properties for the $\overline{\overline{C}}, \overline{\overline{L}}, \overline{\overline{X}}$, and $\overline{\overline{Z}}$ parameters as we will see. However, first we will derive two general properties of bitransmission lines of type (15) not restricted to the quasi-TEM analysis. First we examine the properties of lossless bitransmission lines. The set (15) will be lossless if $\Re(d\overline{I}^T\overline{\phi}^*/dz) = 0$ for all $\overline{I}(z)$ and $\overline{\phi}(z)$, with '$\Re$' the real part of, 'T' the transposition operator and '$*$' the complex conjugate operator. If this condition is elaborated using the relations (15) one finds after some simple mathematics

$$\overline{\overline{C}}^* = \overline{\overline{C}}^T \quad \overline{\overline{L}}^* = \overline{\overline{L}}^T \quad \overline{\overline{X}}^* = \overline{\overline{Z}}^T. \tag{16}$$

This generalizes the result of one line in [15]. It is easy to show with (16) that the propagation coefficients β of the eigenmodes in a lossless set (15) are real as expected. Since we assumed lossless materials in the quasi-TEM analysis the parameters resulting from these analysis will always satisfy (16).

Second we examine the properties of a reciprocal set of transmission lines. A set of transmission lines is reciprocal when

$$\frac{d}{dz}[\overline{I}^T(z)\tilde{\overline{\phi}}(z) - \overline{\phi}^T(z)\tilde{\overline{I}}(z)] = 0 \tag{17}$$

where $\overline{I}(z), \overline{\phi}(z)$ and $\tilde{\overline{I}}(z), \tilde{\overline{\phi}}(z)$ are two arbitrary independent solutions of the set (15). If (17) is elaborated using (15) one easily finds

$$\overline{\overline{C}} = \overline{\overline{C}}^T \quad \overline{\overline{L}} = \overline{\overline{L}}^T \quad \overline{\overline{X}} = -\overline{\overline{Z}}^T. \tag{18}$$

Hence for a lossless reciprocal set of bitransmission lines it follows that $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are real and that $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ are imaginary. In general the coupled bitransmission lines (15) are nonbidirectional [24] which means that eigenmodes propagating in opposite directions in general have different propagation coefficients. However for a reciprocal set it is easy to show, using (18) and some properties of determinants, that

$$\begin{aligned}&\det\begin{pmatrix}\overline{\overline{Z}} - \beta\overline{\overline{I}} & \overline{\overline{L}}\\ \overline{\overline{C}} & \overline{\overline{X}} - \beta\overline{\overline{I}}\end{pmatrix} = 0\\ \Leftrightarrow\ &\det\begin{pmatrix}\overline{\overline{Z}} + \beta\overline{\overline{I}} & \overline{\overline{L}}\\ \overline{\overline{C}} & \overline{\overline{X}} + \beta\overline{\overline{I}}\end{pmatrix} = 0\end{aligned} \tag{19}$$

where $\overline{\overline{I}}$ is the N by N unit matrix. Since the determinant equations in (19) are the eigenvalue equations for the propagation coefficients β and $-\beta$ respectively, (19) proves that reciprocal bitransmission lines are bidirectional. This shows that for bitransmission lines the term bidirectional and reciprocal essentially have the same meaning.

Now we derive a few properties of the $\overline{\overline{C}}, \overline{\overline{L}}, \overline{\overline{X}}$ and $\overline{\overline{Z}}$ parameters which are strictly related to the quasi-TEM analysis behind them. First assume that the whole cross-section consists of pure nonreciprocal material, i.e. that $\kappa(\boldsymbol{r}) = 0$ in the whole cross-section. Then the potential problem (6), (7) is purely real and hence its solution $\overline{\overline{a}}, \overline{\overline{b}}, \overline{\overline{c}}$ and $\overline{\overline{d}}$ is real and of course $\overline{\overline{C}}, \overline{\overline{L}}, \overline{\overline{X}}$ and $\overline{\overline{Z}}$ are also real. And due to (16) this means that $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are symmetric matrices and that $\overline{\overline{X}} = \overline{\overline{Z}}^T$. Hence, the set of transmission lines is also nonreciprocal.

Assume that we know the solution $\overline{\overline{C}}, \overline{\overline{L}}, \overline{\overline{X}}$ and $\overline{\overline{Z}}$ for given $\varepsilon(\boldsymbol{r}), \mu(\boldsymbol{r}), \xi(\boldsymbol{r})$ and $\zeta(\boldsymbol{r})$ then there is an easy relation with the solution for $\varepsilon' = \varepsilon, \mu' = \mu, \xi' = -j\xi$ and $\zeta' = j\zeta$ which is denoted by primes. To see this let us first assume that $\phi_k \neq 0$ and that $\phi_j = \psi_i = 0$ $(i, j = 1, 2, \cdots, N$ and $j \neq k)$ in both the original and the associated problem. If now (7) and (8) are compared for both problems one finds that $\phi'(\boldsymbol{r}) = \phi(\boldsymbol{r}), \psi'(\boldsymbol{r}) = j\psi(\boldsymbol{r}), J'_{z,0} = jJ_{z,0}$ and $\rho'_0 = \rho_0$ and hence that $\overline{I}' = j\overline{I}$ and $\overline{Q}' = \overline{Q}$. From (9) it then follows that $c'_{jk} = jc_{jk}$ and $a'_{jk} = a_{jk}$. Analogously one can show that $b'_{jk} = -jc_{jk}$ and $d'_{jk} = d_{jk}$ assuming that $\psi_k \neq 0$ and that $\phi_i = \psi_j = 0$ $(i, j = 1, 2, \cdots, N$ and $j \neq k)$. With (11) it is then easily proved that

$$\overline{\overline{C}}' = \overline{\overline{C}} \quad \overline{\overline{L}}' = \overline{\overline{L}} \quad \overline{\overline{X}}' = -j\overline{\overline{X}} \quad \overline{\overline{Z}}' = j\overline{\overline{Z}}. \tag{20}$$

Applying (20) twice (to go from ξ to $-j\xi$ and from $-j\xi$ to $-\xi$ and at the same time from ζ to $j\zeta$ and from $j\zeta$ to $-\zeta$) shows that both $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ change sign when both ξ and ζ or both χ and κ change sign.

The result (20) has an interesting consequence. Assume that the whole cross-section of the structure consists of pure chiral or reciprocal material, i.e. that $\chi(\boldsymbol{r}) = 0$ in the whole cross-section. Then, using the technique of previous section, we can transform this problem to a pure nonreciprocal problem treated above. With (20) we then can conclude that $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are real and symmetric matrices and that $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ are imaginary with $\overline{\overline{X}} = -\overline{\overline{Z}}^T$. This means that the condition (18) is satisfied or that the set of transmission lines is reciprocal for reciprocal materials. The transition to the nonreciprocal case allows us also to treat the chiral problem as a pure real problem.

Often the cross section of the structure, like for a microstrip line, contains a symmetry axis. In this case the three-dimensional structure is invariant under a rotation over 180° around that symmetry axis taken in one arbitrary cross-section, for example the section at $z = 0$. By rotating the structure $\overline{Q}(z = 0)$ and $\overline{\phi}(z = 0)$ remain the same and $\overline{I}(z = 0)$ and $\overline{\psi}(z = 0)$ change sign. From (10) it then follows that both $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ vanish. This same conclusion can also be obtained from (15) by demanding that if $\beta, \overline{\phi}, \overline{I}$ is an eigenmode that also $-\beta, \overline{\phi}, -\overline{I}$ is an eigenmode. In this important special case the structure is always represented by a classical set of telegraphers equations with only $\overline{\overline{C}}$ and $\overline{\overline{L}}$ different from zero.

It is interesting to look at the case where the cross-section is homogeneous, i.e. where $\varepsilon(\boldsymbol{r}), \mu(\boldsymbol{r}), \xi(\boldsymbol{r})$ and $\zeta(\boldsymbol{r})$ are independent of $\boldsymbol{r}$. In this case (7) reduces to two classical Laplace equations $\nabla^2_{\text{tr}}\phi(\boldsymbol{r}) = 0$ and $\nabla^2_{\text{tr}}\psi(\boldsymbol{r}) = 0$ which can be solved independently. Now it is easy to show that if $\overline{\overline{C}}_v$ is the vacuum $(\varepsilon(\boldsymbol{r}) = \varepsilon_0)$ capacitance matrix of the structure that $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are given by:

$$\overline{\overline{C}} = \frac{\varepsilon_r\mu_r - \chi^2 - \kappa^2}{\mu_r}\overline{\overline{C}}_v \quad \overline{\overline{L}} = \frac{\mu_r}{c^2}\overline{\overline{C}}_v^{-1} \tag{21}$$

with $c = 1/\sqrt{\varepsilon_0\mu_o}$ the speed of light in vacuum. The bi-isotropy only influences the capacitance matrix.

The two previous paragraphs show that, although the structure consists of nonreciprocal material, it is not necessary that the corresponding bitransmission line is nonreciprocal. This shows that a waveguide consisting of nonreciprocal materials is not necessary nonbidirectional in accordance with [24].

Finally we point out a discrepancy with the theory of [15]. In [15] only a single bitransmission line is investigated and X and Z (which are now just scalars) are respectively written as $a+jb$ and $a-jb$. The quantity a was called chirality parameter and the quantity b nonreciprocity or Tellegen parameter. From the pure chiral and pure nonreciprocal case handled above we found a result which is just the opposite of this, i.e. a is due to nonreciprocity and b due to chirality. Moreover from the general circuit concept for reciprocity (18) it follows that a bitransmission line with $b = 0$ and $a \neq 0$ is nonreciprocal and that a line with $b \neq 0$ and $a = 0$ is reciprocal. The reason for this, in our opinion, unfortunate nomenclature of a and b in [15] finds its origin in the application of the bitransmission line model to the propagation of circulary polarized plane waves in layered bi-isotropic media. The example handled in [15] indeed gives a nonreciprocal bitransmission line for a reciprocal medium because only one orientation of polarization in each propagation direction was taken into account as has been addressed at the end of [15].

V. Generalization to Bianisotropic Media

In this section the generalization to general inhomogeneous bianisotropic materials is discussed and it is shown for which bianisotropic materials the theory reduces to the bi-isotropic case discussed above. For bianisotropic materials the material parameters in (1) become dyadics. Equation (2) now becomes

$$\begin{aligned} \boldsymbol{D} &= \overline{\overline{T}}(\boldsymbol{r})\boldsymbol{E} + \overline{\overline{U}}(\boldsymbol{r})\boldsymbol{B} \\ \boldsymbol{H} &= \overline{\overline{V}}(\boldsymbol{r})\boldsymbol{E} + \overline{\overline{W}}(\boldsymbol{r})\boldsymbol{B} \end{aligned} \tag{22}$$

with

$$\begin{aligned} \overline{\overline{T}} &= \overline{\overline{\varepsilon}} - \overline{\overline{\xi}}\,\overline{\overline{\mu}}^{-1}\overline{\overline{\zeta}} \qquad & \overline{\overline{U}} &= \overline{\overline{\xi}}\,\overline{\overline{\mu}}^{-1} \\ \overline{\overline{V}} &= -\overline{\overline{\mu}}^{-1}\overline{\overline{\zeta}} & \overline{\overline{W}} &= \overline{\overline{\mu}}^{-1}. \end{aligned} \tag{23}$$

It is assumed that the material defined in (22) is lossless which means that $\overline{\overline{\varepsilon}}^T = \overline{\overline{\varepsilon}}^*, \overline{\overline{\mu}}^T = \overline{\overline{\mu}}^*$ and $\overline{\overline{\zeta}}^T = \overline{\overline{\zeta}}^*$ (see [16]). For a general material of type (22) we only have that $\beta_0 = E_{z,0} = H_{z,0} = 0$ and that $\boldsymbol{J}_{\text{tr},0} = \boldsymbol{0}$ but not that $D_{z,0}$ and $B_{z,0}$ are zero. This means that we have the following relation between the zeroth order transversal components in (22) after eliminating $B_{z,0}$:

$$\begin{aligned} \boldsymbol{D}_{\text{tr},0} &= \overline{\overline{T}}'(\boldsymbol{r})\boldsymbol{E}_{\text{tr},0} + \overline{\overline{U}}'(\boldsymbol{r})\boldsymbol{B}_{\text{tr},0} \\ \boldsymbol{H}_{\text{tr},0} &= \overline{\overline{V}}'(\boldsymbol{r})\boldsymbol{E}_{\text{tr},0} + \overline{\overline{W}}'(\boldsymbol{r})\boldsymbol{B}_{\text{tr},0} \end{aligned} \tag{24}$$

with

$$\begin{aligned}\overline{\overline{T}}' &= \overline{\overline{T}}_{\rm trtr} + \frac{\overline{U}_{\rm trz}\overline{V}^T_{\rm ztr}}{W_{zz}} \qquad \overline{\overline{U}}' = \overline{\overline{U}}_{\rm trtr} + \frac{\overline{U}_{\rm trz}\overline{W}^T_{\rm ztr}}{W_{zz}}\\ \overline{\overline{V}}' &= \overline{\overline{V}}_{\rm trtr} + \frac{\overline{W}_{\rm trz}\overline{V}^T_{\rm ztr}}{W_{zz}} \qquad \overline{\overline{W}}' = \overline{\overline{W}}_{\rm trtr} + \frac{\overline{W}_{\rm trz}\overline{W}^T_{\rm ztr}}{W_{zz}}\end{aligned} \tag{25}$$

where we have decomposed the four dyadics as

$$\overline{\overline{P}} = \begin{pmatrix} \overline{\overline{P}}_{\rm trtr} & \overline{P}_{\rm trz} \\ \overline{P}^T_{\rm ztr} & P_{zz} \end{pmatrix} \tag{26}$$

with $P = T, U, V$, or W. Note that in (25) $\overline{U}_{{\rm tr},z}\overline{V}^T_{\rm ztr}$ for example denotes a transversal 2×2 dyad and not a scalar product of two vectors. Equations (4) and (6) remain unchanged but the potentials ϕ and Ψ now satisfy the following equations:

$$\begin{aligned} -\nabla_{\rm tr} \cdot \overline{\overline{T}}' \cdot \nabla_{\rm tr}\phi + \nabla_{\rm tr} \cdot \overline{\overline{U}}' \cdot (\nabla_{\rm tr} \times \psi \boldsymbol{u}_z) &= 0\\ \boldsymbol{u}_z \cdot [-\nabla_{\rm tr} \times \overline{\overline{V}}' \cdot \nabla_{\rm tr}\phi + \nabla_{\rm tr} \times \overline{\overline{W}}' \cdot (\nabla_{\rm tr} \times \psi \boldsymbol{u}_z)] &= 0. \end{aligned} \tag{27}$$

At the conductors the boundary conditions (8) have to be replaced by

$$\phi = \text{constant} \qquad \psi = \text{constant}$$

$$\begin{aligned} \boldsymbol{u}_n \cdot \overline{\overline{T}}' \cdot \boldsymbol{u}_n \frac{\partial \phi}{\partial n} - \boldsymbol{u}_n \cdot \overline{\overline{U}}' \cdot \boldsymbol{u}_t \frac{\partial \psi}{\partial n} &= \rho_0\\ \boldsymbol{u}_t \cdot \overline{\overline{V}}' \cdot \boldsymbol{u}_n \frac{\partial \phi}{\partial n} - \boldsymbol{u}_t \cdot \overline{\overline{W}}' \cdot \boldsymbol{u}_t \frac{\partial \psi}{\partial n} &= J_{z,0} \end{aligned} \tag{28}$$

where $\boldsymbol{u}_t = \boldsymbol{u}_z \times \boldsymbol{u}_n$. Since the problem is still linear (9) and hence (10) remain valid and also (12) to (15) remain valid. It is interesting to remark that in the pure anisotropic case ($\overline{\overline{\xi}} = \overline{\overline{\zeta}} = 0$) the potential problems (27), (28) for ϕ and ψ are decoupled and that $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ are zero since $\overline{\overline{U}}'$ and $\overline{\overline{V}}'$ are zero.

It can be verified that the problem (27), (28) reduces to the bi-isotropic case when $\overline{\overline{T}}', \overline{\overline{U}}', \overline{\overline{V}}'$ and $\overline{\overline{W}}'$ are multiples of the unit dyadic. In fact even when $\overline{\overline{T}}', \overline{\overline{U}}', \overline{\overline{V}}'$ and $\overline{\overline{W}}'$ are multiples of the same arbitrary 2×2 dyadic one can also reduce the problem to the bi-isotropic case, by means of an affine transformation (see [16]). This means that if one can solve the bi-isotropic case one can also solve several bianisotropic cases. An important special case are uniaxial materials with the axis along the propagation direction z.

VI. Conclusion

The quasi-TEM analysis for multiconductor lines in inhomogeneous isotropic media was extended to bi-isotropic media. It was shown that in the quasi-TEM limit the multiconductor line can be represented in circuit terms by so-called coupled bitransmission lines. These bitransmission lines are characterized by four circuit matrices $\overline{\overline{C}}, \overline{\overline{L}}, \overline{\overline{X}}$ and $\overline{\overline{Z}}$. It was further shown that for lossless bitransmission lines $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are Hermitian ($\overline{\overline{C}}^* = \overline{\overline{C}}^T$ and $\overline{\overline{L}}^* = \overline{\overline{L}}^T$) and that $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ are Hermitian conjugates ($\overline{\overline{X}}^* = \overline{\overline{Z}}^T$). For reciprocal bitransmission lines it was shown that $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are symmetric and that $\overline{\overline{X}}$ is minus the transposed of $\overline{\overline{Z}}$. For pure chiral media ($\chi = 0$) it was shown that $\overline{\overline{C}}$ and $\overline{\overline{L}}$ are real and that $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ are imaginary. In the pure nonreciprocal case ($\kappa = 0$) $\overline{\overline{C}}$ and $\overline{\overline{L}}$ remain real and $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ also become real. Finally it was shown that $\overline{\overline{X}}$ and $\overline{\overline{Z}}$ vanish for structures with a symmetry axis in the cross-section. In the last section the quasi-TEM analysis was generalized to bianisotropic media.

References

[1] F. Mariotte and J. P. Parneix, Eds., in *Proc. Chiral '94 Symp. (URSI-IEEE)*, Périgueux, France, May 1994.

[2] F. Olyslager and D. De Zutter, "Rigourous full-wave analysis of electric and dielectric waveguides embedded in a multilayered bianisotropic medium," *Radio Sci.*, vol. 28, pp. 937–946, Sept./Oct. 1993.

[3] D. M. Pozar, "Microstrip antennas and arrays on chiral substrates," *IEEE Trans. Antenna. Propagat.*, vol. 40, no. 10, pp. 1260–1263, Oct. 1992.

[4] J. L. Tsalamengas, "Interaction of electromagnetic waves with general bianisotropic slabs," *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 10, pp. 1870–1878, Oct. 1992.

[5] F. Olyslager, B. Baekelandt, D. De Zutter, and M. Van Craenendonck, "Plane wave scattering at structures consisting of bianisotropic layers and resistive sheets," in *Proc. Joint 3rd Int. Conf. Electromagn. in Aerospace Applicat. and 7th Euro. Electromagn. Structures Conf*, Torino, Italy, pp. 295–298, Sept. 1993.

[6] N. Faché, F. Olyslager, and D. De Zutter, *Electromagnetic and Circuit Modelling of Multiconductor Lines.* Oxford, U.K.: Clarendon, Mar. 1993.

[7] I. V. Lindell, "On the quasi-TEM modes in inhomogeneous multiconductor transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, no. 8, pp. 812–817, Aug. 1981.

[8] I. V. Lindell and Q. Gu, "Theory of time-domain quasi-TEM modes in inhomogeneous multiconductor transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no. 10, pp. 893–897, Oct. 1987.

[9] C. Wei, R. F. Harrington, J. R. Mautz, and T. K. Sarkar, "Multiconductor transmission lines in multilayered media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, no. 4, pp. 439–450, Apr. 1984.

[10] F. Olyslager, N. Faché, and D. De Zutter, "New fast and accurate line parameter calculation of multiconductor transmission lines in multilayered media," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 6, pp. 901–909, June 1991.

[11] M. Horno, F. L. Mesa, F. Medina, and R. Marques, "Quasi-TEM analysis of multilayered, multiconductor coplanar structures with dielectric and magnetic anisotropy including substarte losses," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 8, pp. 1059–1068, Aug. 1990.

[12] T. Dhaene, F. Olyslager, and D. De Zutter, "Circuit modeling of general hybrid uniform waveguide structures in chiral anisotropic inhomogeneous media," *Analog Integrated Circuits and Signal Processing, Special Issue on High-Speed Interconnects.* Norwell, MA: Kluwer, vol. 5, 1994, pp. 57–66.

[13] F. Olyslager, "A reciprocity based generalized circuit model for hybrid bi-isotropic waveguide structures," in *Proc. 1994 IEEE Antenna. Propagat. Symp.*, Seattle, WA, June 1994, pp. 2254–2257.

[14] P. K. Koivisto and J. C.-E. Sten, "Quasi-static image method applied to bi-isotropic microstrip geometry," accepted for publication in *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 1, pp. 169–175, Jan. 1995.

[15] I. V. Lindell, M. E. Valtonen, and A. H. Sihvola, "Theory of nonreciprocal and nonsymmetric uniform transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 2, pp. 291–297, Feb. 1994.

[16] I. V. Lindell, *Methods for Electromagnetic Field Analysis.* Oxford, U.K.: Clarendon, 1992.

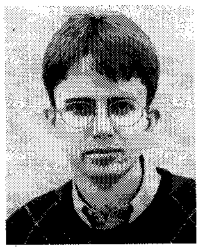
[17] F. L. Mesa, G. Cano, F. Medina, R. Marques, and M. Horno, "On the quasi-TEM and full-wave approaches applied to coplanar multistrip on lossy dielectric layered media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-40, no. 3, pp. 524–531, Mar. 1992.

[18] R. F. Harrington and C. Wei, "Losses on multiconductor transmission lines in multilayered dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, no. 7, pp. 705–710, July 1984.

[19] R. Marques and M. Horno, "Propagation of quasi-static modes in anisotropic transmission lines: Application to MIC lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, no. 10, pp. 927–932, Oct. 1985.

[20] A. Lakhtakia and W. Weiglhofer, "Are linear, nonreciprocal biisotropic media forbidden?" *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 9, pp. 1715–1716, Sept. 1994.

[21] F. Olyslager, E. Laermans, and D. De Zutter, "Rigorous quasi-TEM analysis of multiconductor transmission lines in bi-isostropic media—Part II: Numerical solution for layered media," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 7, pp. 1416–1423, July 1995.
[22] F. Olyslager and E. Laermans, "Fast and accurate line parameter calculation of general multiconductor transmission lines embedded in multilayered bi-isotropic media," in *Proc. PIERS 1994 Symp.*, Noordwijk, The Netherlands, pp. 130, July 1994.
[23] E. J. Post, *Formal Structure of Electromagnetics.* Amsterdam: North Holland, 1962.
[24] P. R. McIsaac, "Mode orthogonality in reciprocal and nonreciprocal waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 11, pp. 1808–1816, Nov. 1991.



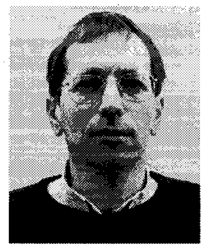
Frank Olyslager (S'90–M'94) was born in Wilrijk, Belgium, on November 24, 1966. He received the electrical engineering degree from the University of Ghent, Belgium, in July 1989. In February 1993 he received the Ph.D. degree from the Laboratory of Electromagnetism and Acoustics (LEA) of the University of Ghent.

From 1989 to 1993 he was a Research Assistant of the National Fund for Scientific Research of Belgium. He is currently a postdoctoral researcher of the National Fund for Scientific Research of Belgium in the Department of Information Technology (the former LEA) of the University of Ghent. His current research interests are in the use of integral equation techniques to solve Maxwell's equations numerically. His activities focus on the electromagnetic wave propagation along high-frequency electrical and optical interconnections in multilayered isotropic and bianisotropic media, on the singularity of electromagnetic fields at edges and tips, and on the study of Green's dyadics in bianisotropic media. He is also investigating the construction of transmission line models for general waveguide structures and Electromagnetic Compatibility (EMC) problems on Printed Circuit Boards and microwave circuits. He is author or coauthor of more than 35 papers in international journals and conference proceedings. He is also co-author of the book *Electromagnetic and Circuit Modelling of Multiconductor Lines* (Clarendon Press, 1993).

Dr. Olyslager received the 1995 Microwave Prize of the IEEE Microwave Theory and Techniques Society.

Eric Laermans was born in Brussels, Belgium, on September 27, 1971. He received the degree in physical engineering from the University of Ghent in July 1994.

At present he is working toward the Ph.D. degree in physical engineering at the Department of Information Technology of the University of Ghent as a Research Assistant of the National Fund for Scientific Research of Belgium. His current research interests are in EMC problems and statistical analysis of electromagnetic problems.



Daniël De Zutter (M'92) was born in Eeklo, Belgium, on November 8, 1953. He received the degree in electrical engineering from the University of Gent, Belgium in July 1976. In October 1981 he received the Ph.D. degree and in the spring of 1984 he completed a thesis leading to a degree equivalent to the French Aggrégation or the German Habilitation.

From 1976 to 1984 he was a Research and Teaching Assistant in the Laboratory of Electromagnetism and Acoustics (now the Department of Information Technology) at the same university. He is now a Professor in the Department of Information Technology, University of Gent and Research Director of the National Fund for Scientific Research of Belgium. Most of his earlier scientific work dealt with the electrodynamics of moving media. His current research now focuses on all aspects of circuit and electromagnetic modelling of high-speed and high-frequency interconnections, on electromagnetic compatibility (EMC) and electromagnetic interference (EMI) topics and on indoor propagation. As author or coauthor he has contributed to about 130 papers in international journals and conference proceedings. He coauthored the book *Electromagnetic and Circuit Modelling of Multiconductor Lines,* (Clarendon Press, 1993).

In 1990 he was elected as a member of the Electromagnetics Society. He received the 1995 Microwave Prize of the IEEE Microwave Theory and Techniques Society.